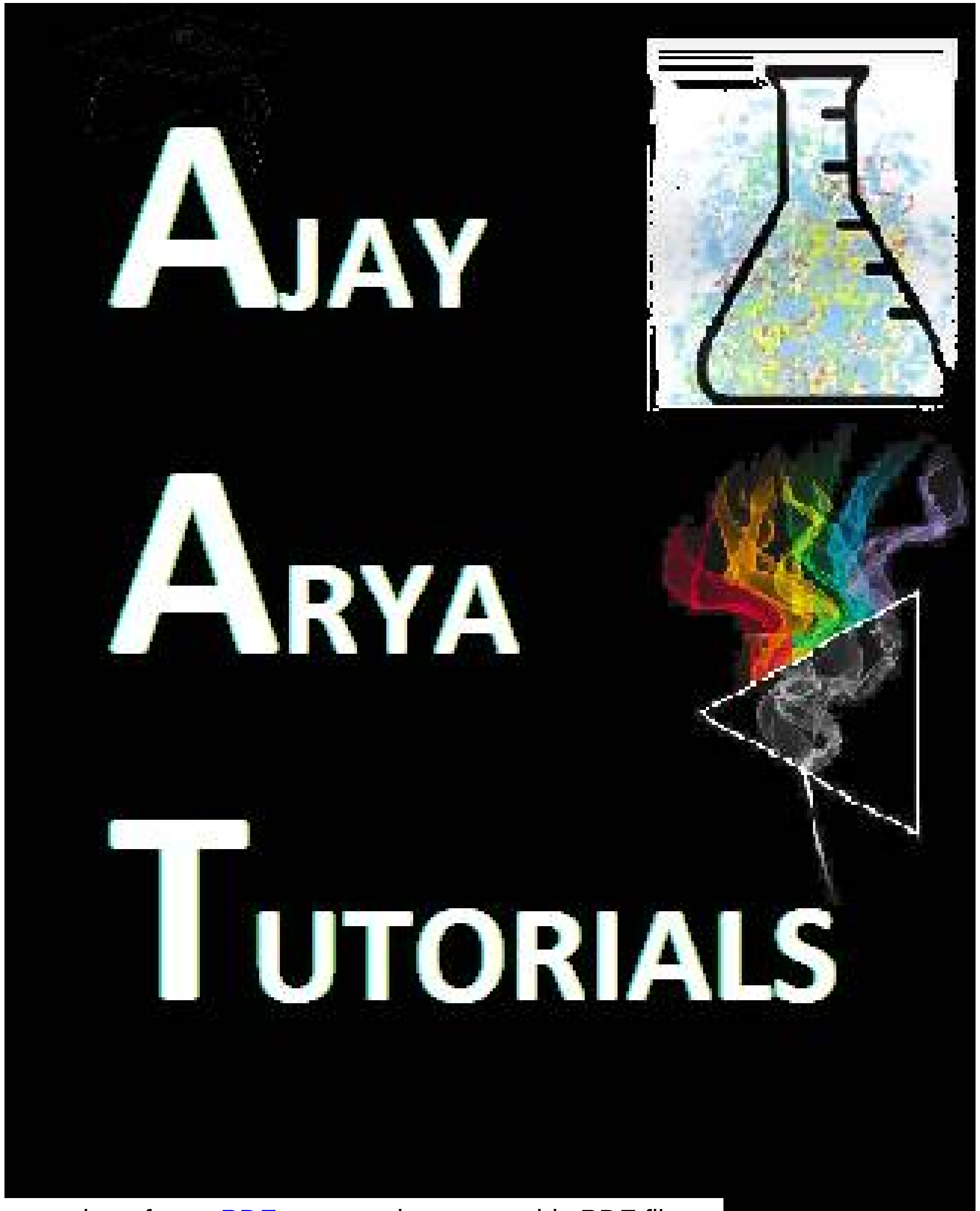
Moving Charges and Magnetism



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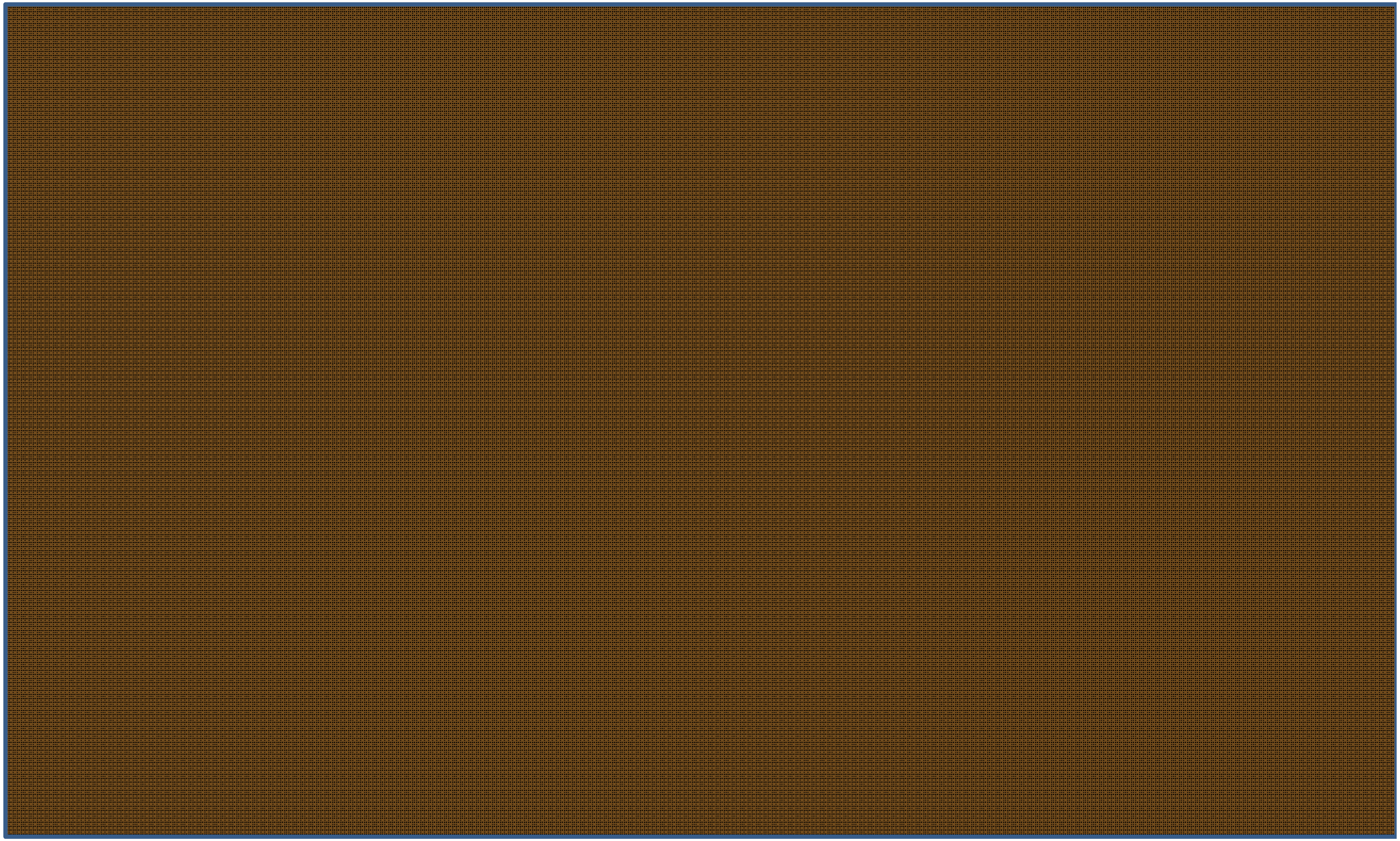


Magnetic Force due to a Magnetic Field

- On a moving charge: $\boldsymbol{F} = q(\boldsymbol{v} \times \boldsymbol{B})$
- On a current-carrying conductor: $\boldsymbol{F} = i(\boldsymbol{l} \times \boldsymbol{B})$

Example 4.1 A straight wire of mass 200 g and length 1.5 m carries a current of 2 A. It is suspended in mid-air by a uniform horizontal magnetic field **B** (Fig. 4.3). What is the magnitude of the magnetic field?

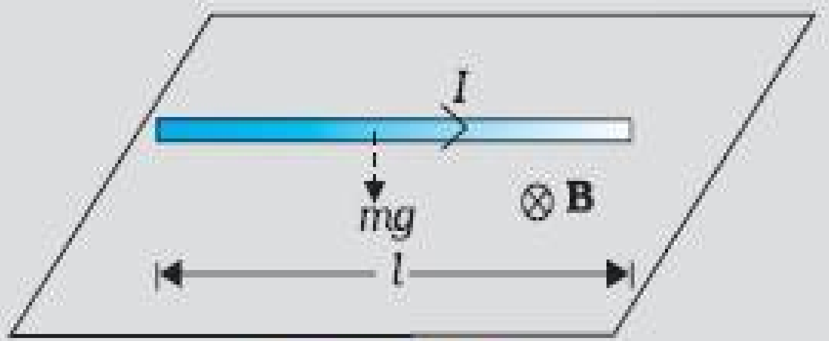


FIGURE 4.3

Hint: Magnetic force is balancing the weight of the wire.

Example 4.2 If the magnetic field is parallel to the positive y-axis and the charged particle is moving along the positive x-axis (Fig. 4.4), which way would the Lorentz force be for (a) an electron (negative charge), (b) a proton (positive charge).

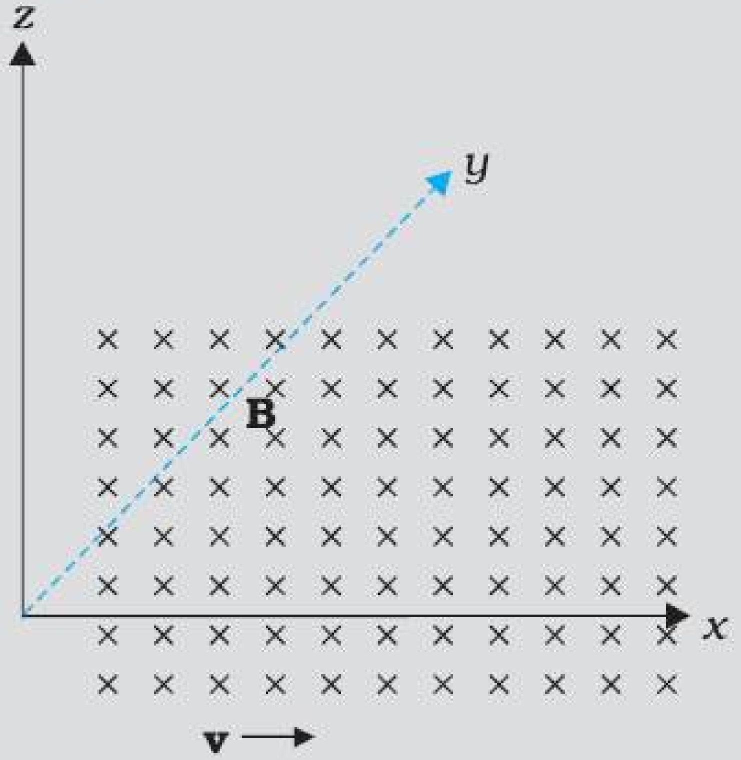
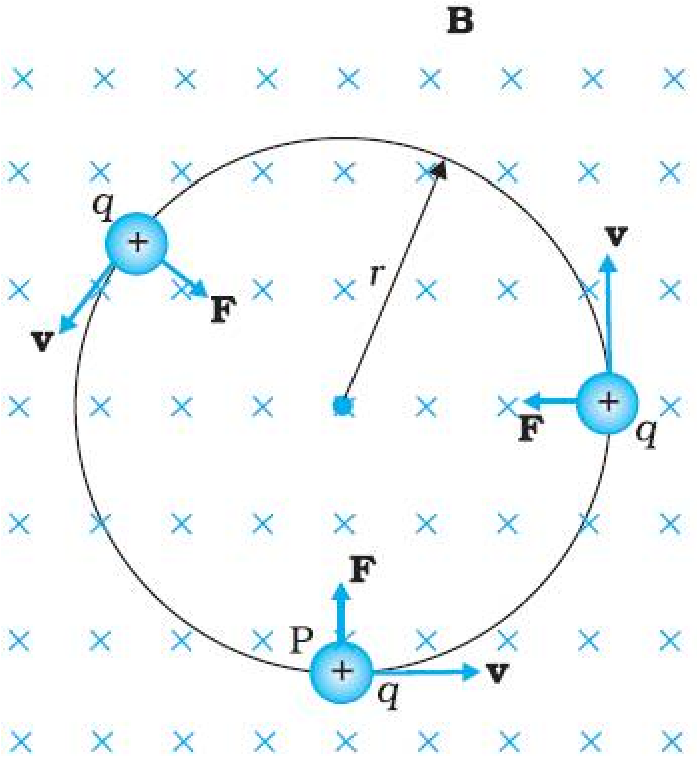


FIGURE 4.4

Motion in a Magnetic Field



Plane Circular Motion if the Field and Velocity are exactly perpendicular.

Radius of Circular Path:

$F = mv^2/R = qvB$
=> **$R = mv/qB$**

Time period of Circular Motion:

$T = 2\pi R/v = 2\pi\ (mv/qB)/v$
=> **$T = 2\pi m/qB$**

Example 4.3 What is the radius of the path of an electron (mass 9×10^{-31} kg and charge 1.6×10^{-19} C) moving at a speed of 3×10^{7} m/s in a magnetic field of 6×10^{-4} T perpendicular to it? What is its frequency? Calculate its energy in keV. (1 eV = 1.6×10^{-19} J).

Motion in a Magnetic Field

Radius of Circular Path:

$F = mv_{\perp}^{2}/R = qvB$

=> $\mathbf{R = mv_{\perp}/qB}$

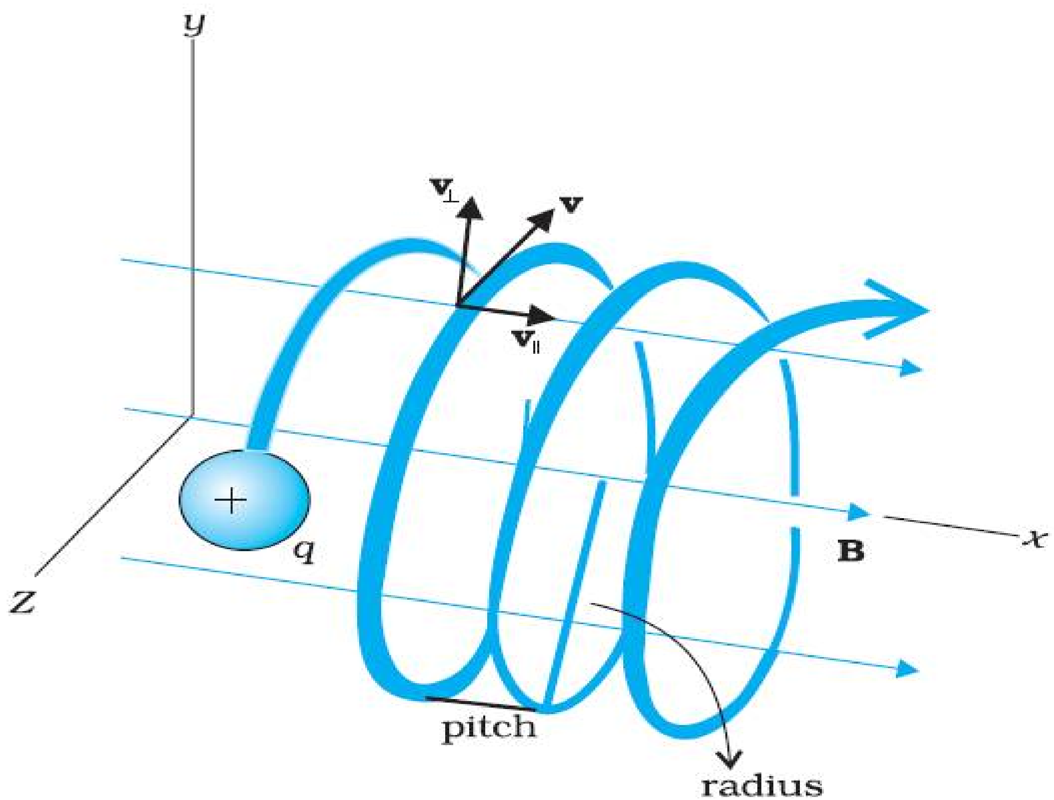
Time period of Circular Motion:

$T = 2\pi R/v = 2\pi\,(mv/qB)/v$
=> $\mathbf{T = 2\pi m/qB}$

Pitch of Helical Motion:

$p = v_{\parallel}T$

=> $\mathbf{p = 2\pi m\, v_{\parallel}/qB}$



Helical Motion if the Field and Velocity are not perpendicular.

Magnetic Field created by an Electric Current

Biot-Savart Law

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{l}\times\mathbf{r}}{r^3}$$

The magnitude of this field is

$$|\mathrm{d}\mathbf{B}| = \frac{\mu_0}{4\pi}\frac{I\,\mathrm{d}l\sin\theta}{r^2}$$

Here, $\frac{\mu_0}{4\pi} = 10^{-7}\,\mathrm{Tm/A}$

Fun Fact: $\varepsilon_0\mu_0 = \frac{1}{c^2}$

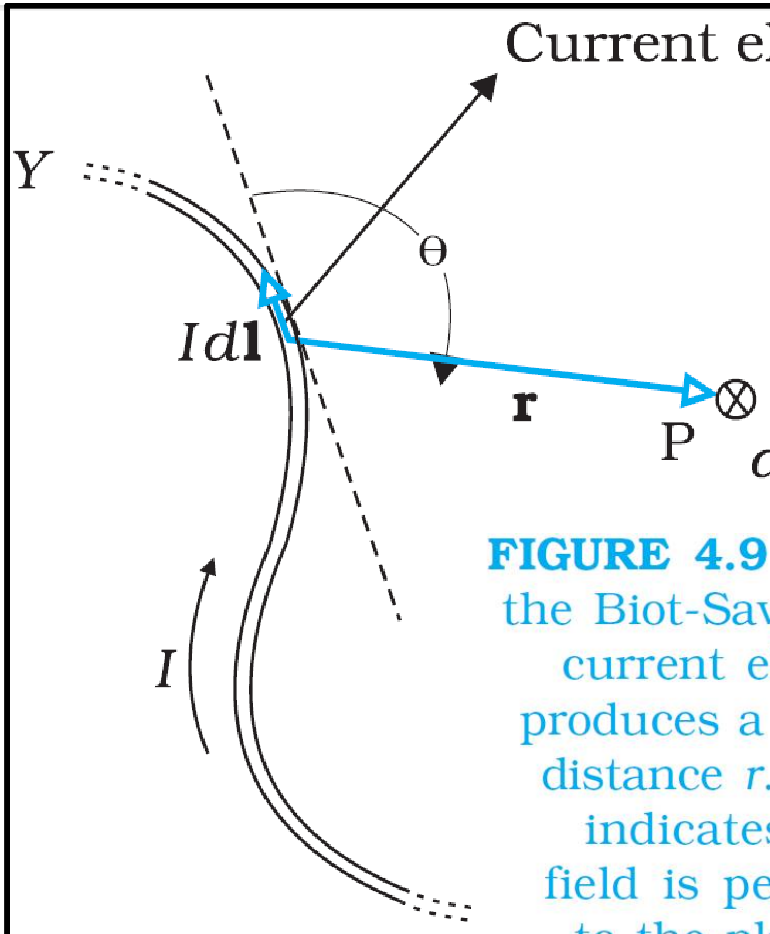
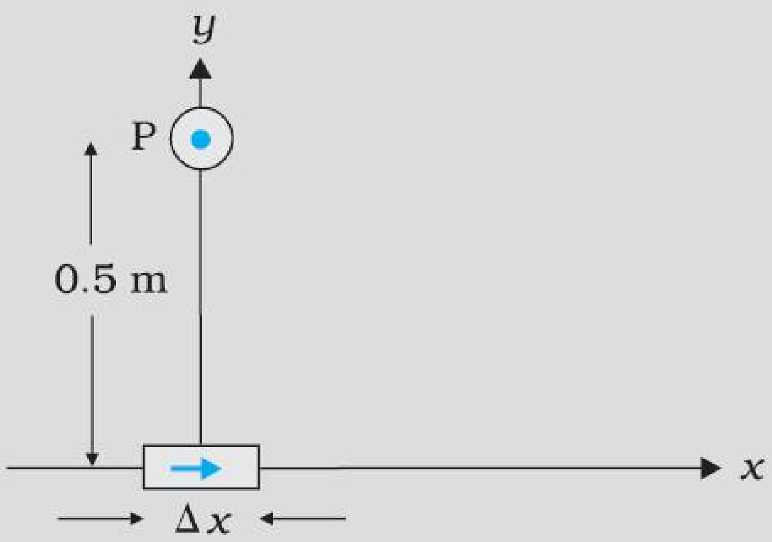
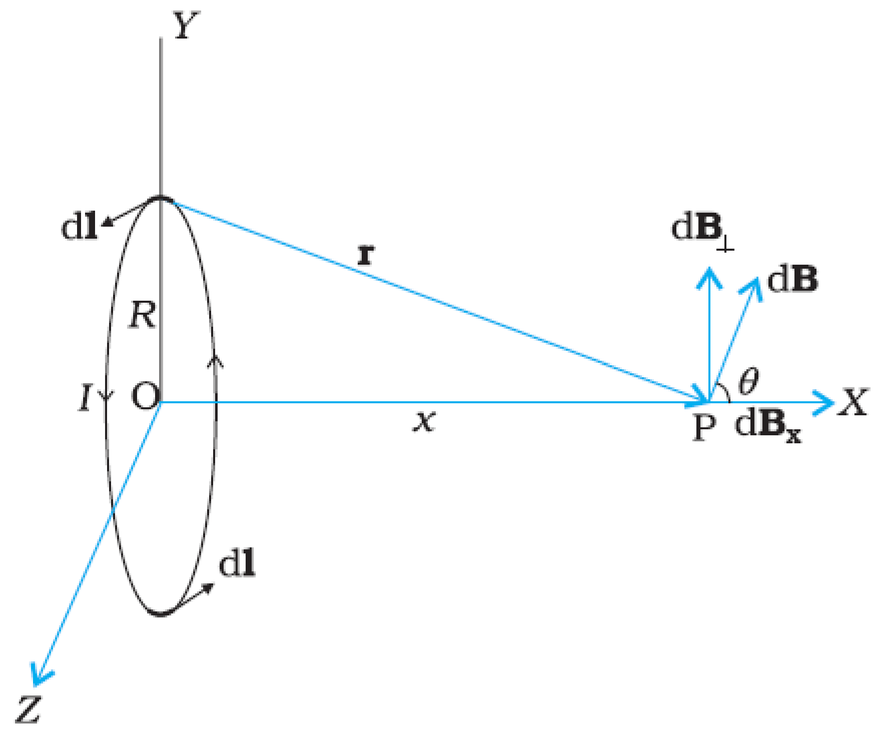


FIGURE 4.9 Illustration of the Biot-Savart law. The current element $I\,\mathrm{d}\mathbf{l}$ produces a field $\mathrm{d}\mathbf{B}$ at a distance r. The ⊗ sign indicates that the field is perpendicular to the plane of this page and directed into it.

Example 4.5 An element $\Delta \mathbf{l} = \Delta x\,\hat{\mathbf{i}}$ is placed at the origin and carries a large current I = 10 A (Fig. 4.10). What is the magnetic field on the y-axis at a distance of 0.5 m. Δx = 1 cm.



Magnetic Field on the Axis of a Circular Current Loop



On the Axis,

$$\mathbf{B} = B_x\hat{\mathbf{i}} = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}\hat{\mathbf{i}}$$

At the center, x = 0 so that

$$\mathbf{B}_0 = \frac{\mu_0 I}{2R}\hat{\mathbf{i}}$$

Direction of Magnetic Field of a circular current loop

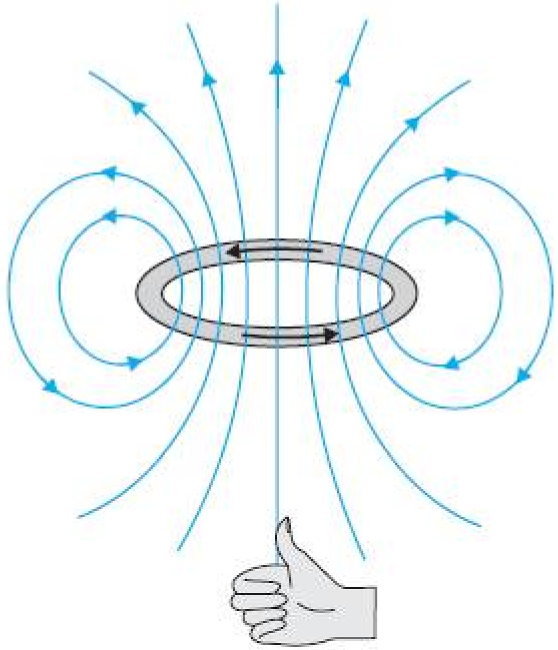
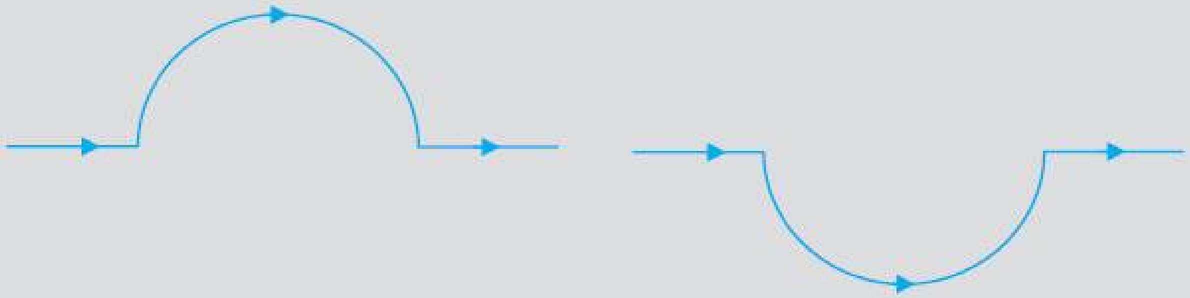


FIGURE 4.12 The magnetic field lines for a current loop. The direction of the field is given by the right-hand thumb rule described in the text. The upper side of the loop may be thought of as the north pole and the lower side as the south pole of a magnet.

Example 4.6 A straight wire carrying a current of 12 A is bent into a semi-circular arc of radius 2.0 cm as shown in Fig. 4.13(a). Consider the magnetic field **B** at the centre of the arc. (a) What is the magnetic field due to the straight segments? (b) In what way the contribution to **B** from the semicircle differs from that of a circular loop and in what way does it resemble? (c) Would your answer be different if the wire were bent into a semi-circular arc of the same radius but in the opposite way as shown in Fig. 4.13(b)?



(a) (b)

FIGURE 4.13

Example 4.7 Consider a tightly wound 100 turn coil of radius 10 cm, carrying a current of 1 A. What is the magnitude of the magnetic field at the centre of the coil?

Ampere’s Circuital Law

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

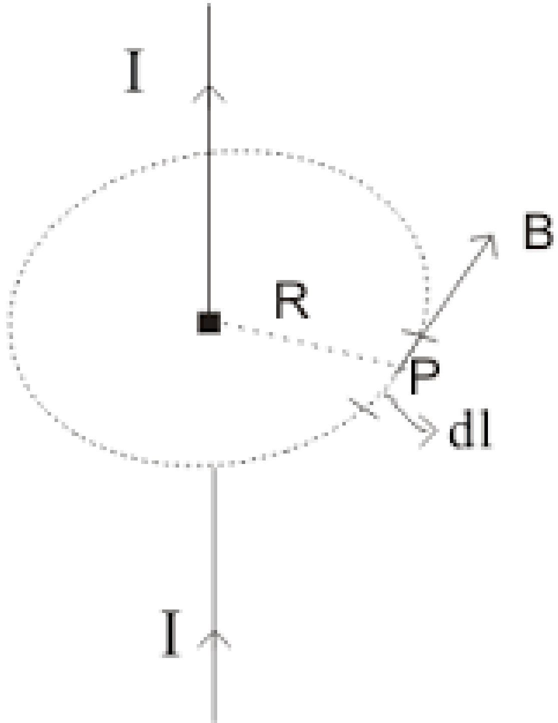
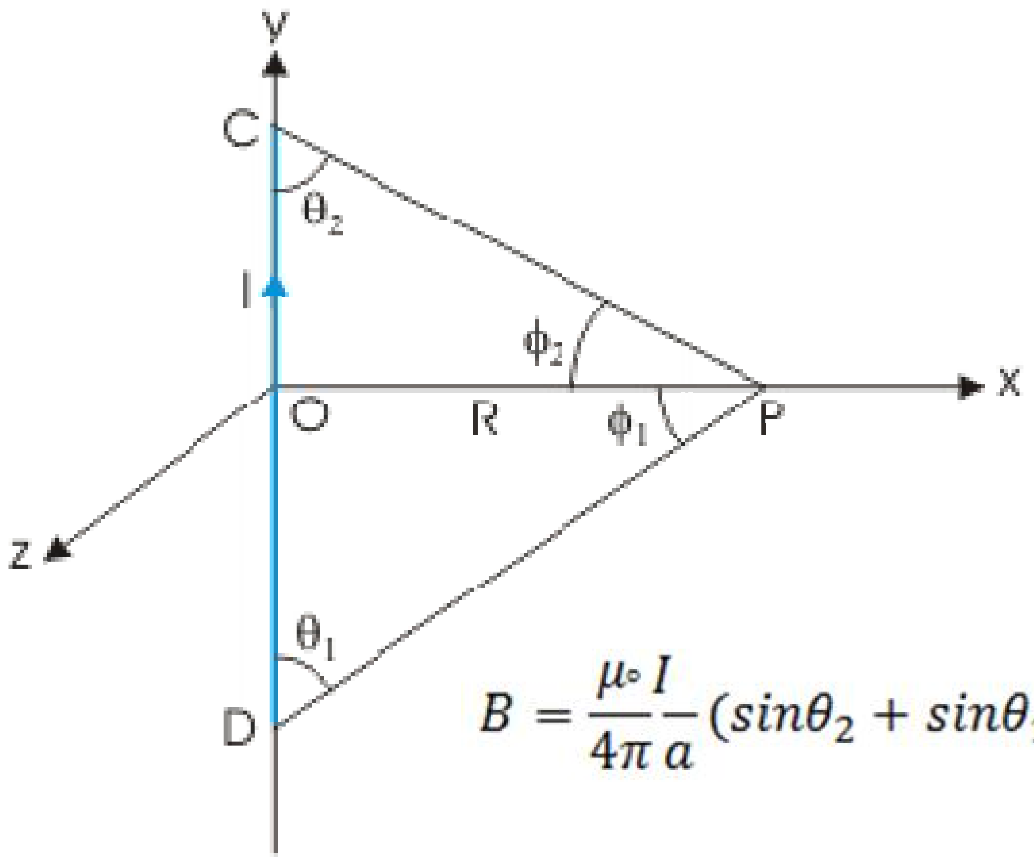


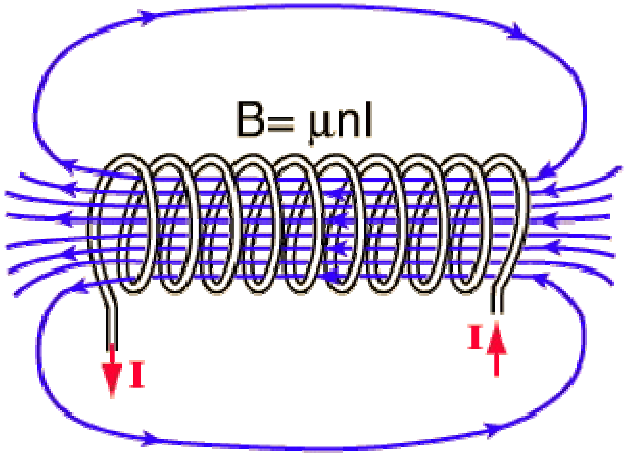
Figure 10. B is the magnetic field due to current carrying conductor at point P

Magnetic Field due to a Straight Current



$$B = \frac{\mu_\circ I}{4\pi\, a}(sin\theta_2 + sin\theta_1)$$

Magnetic Field due to a Solenoid



The magnetic field is concentrated into a nearly uniform field in the center of a long solenoid. The field outside is weak and divergent.

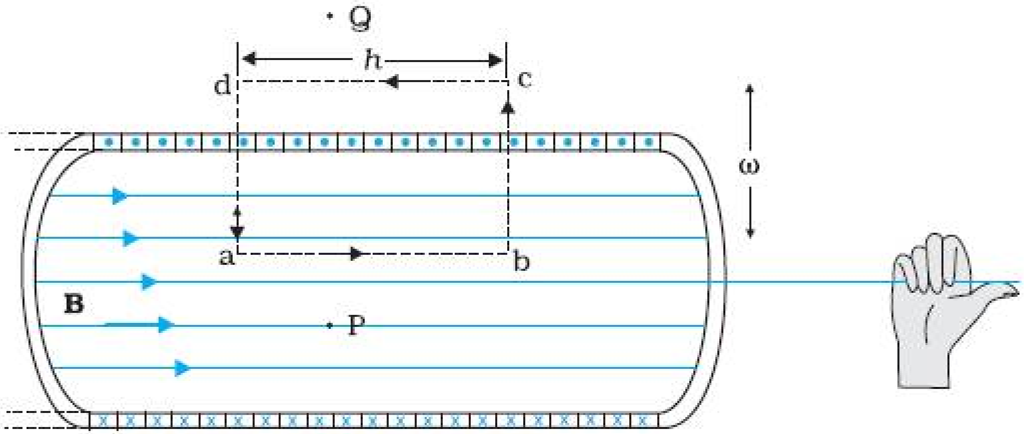
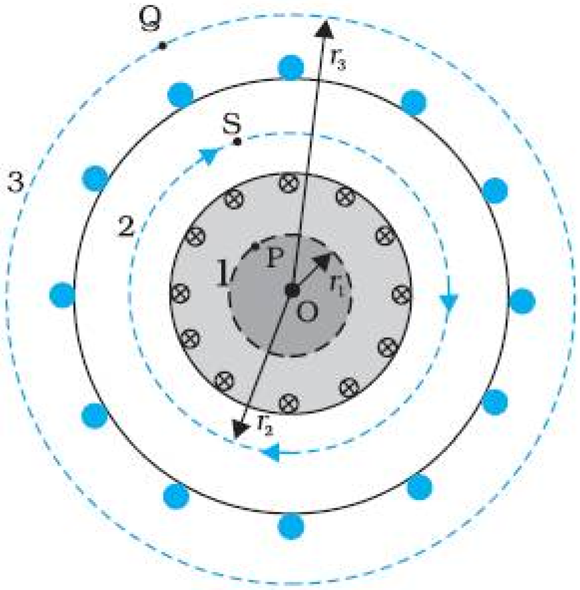


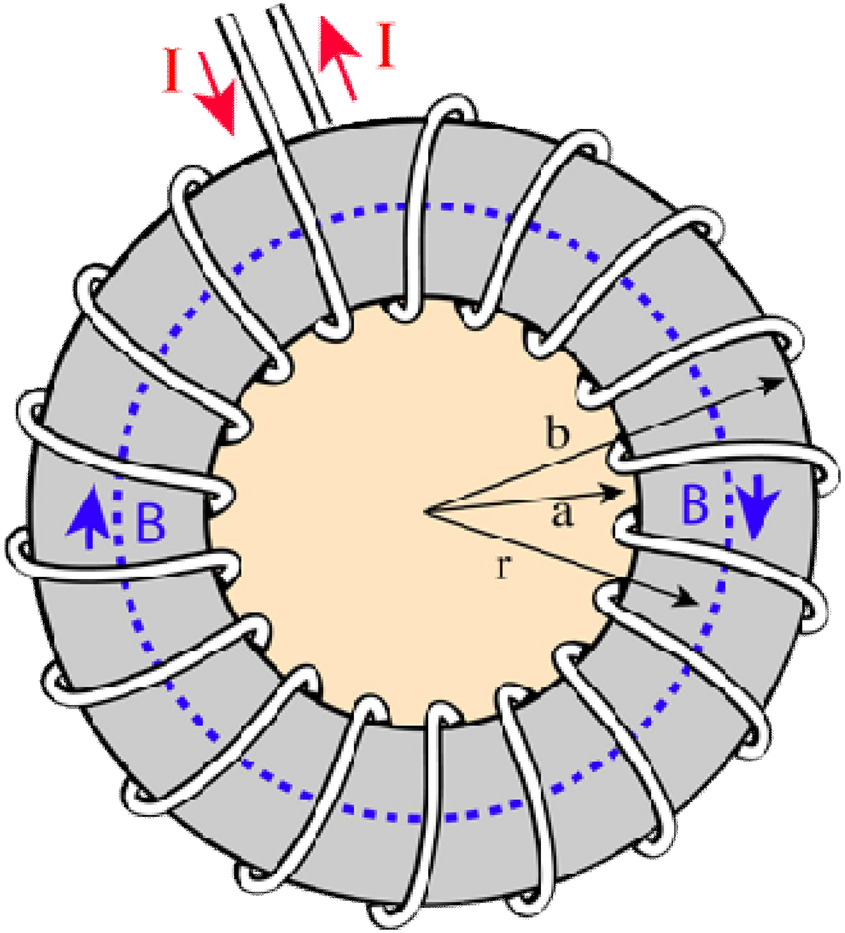
FIGURE 4.18 The magnetic field of a very long solenoid. We consider a rectangular Amperian loop abcd to determine the field.

Magnetic Field due to a Torroid

$$B 2\pi r = \mu N I$$

$$B = \frac{\mu N I}{2\pi r}$$





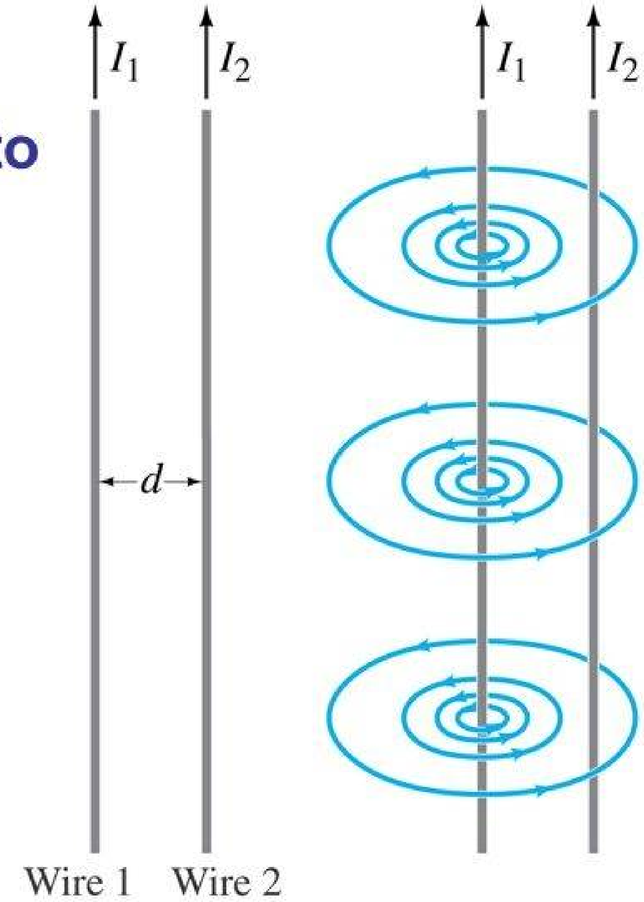
Force between Parallel Currents

The magnetic field produced at the position of wire 2 due to the current in wire 1 is:

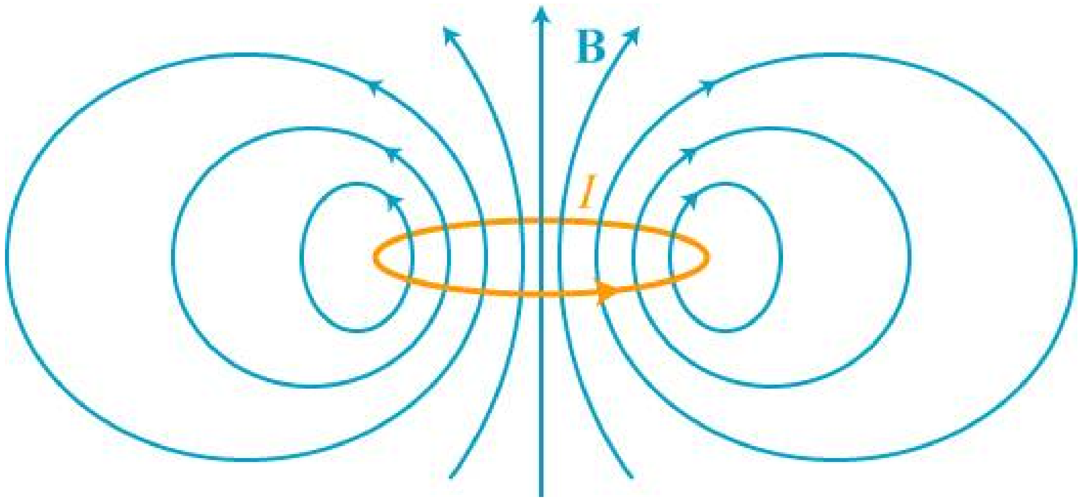
$$B_1 = \frac{\mu_0}{2\pi}\frac{I_1}{d}$$

The force this field exerts on a length l_2 of wire 2 is:

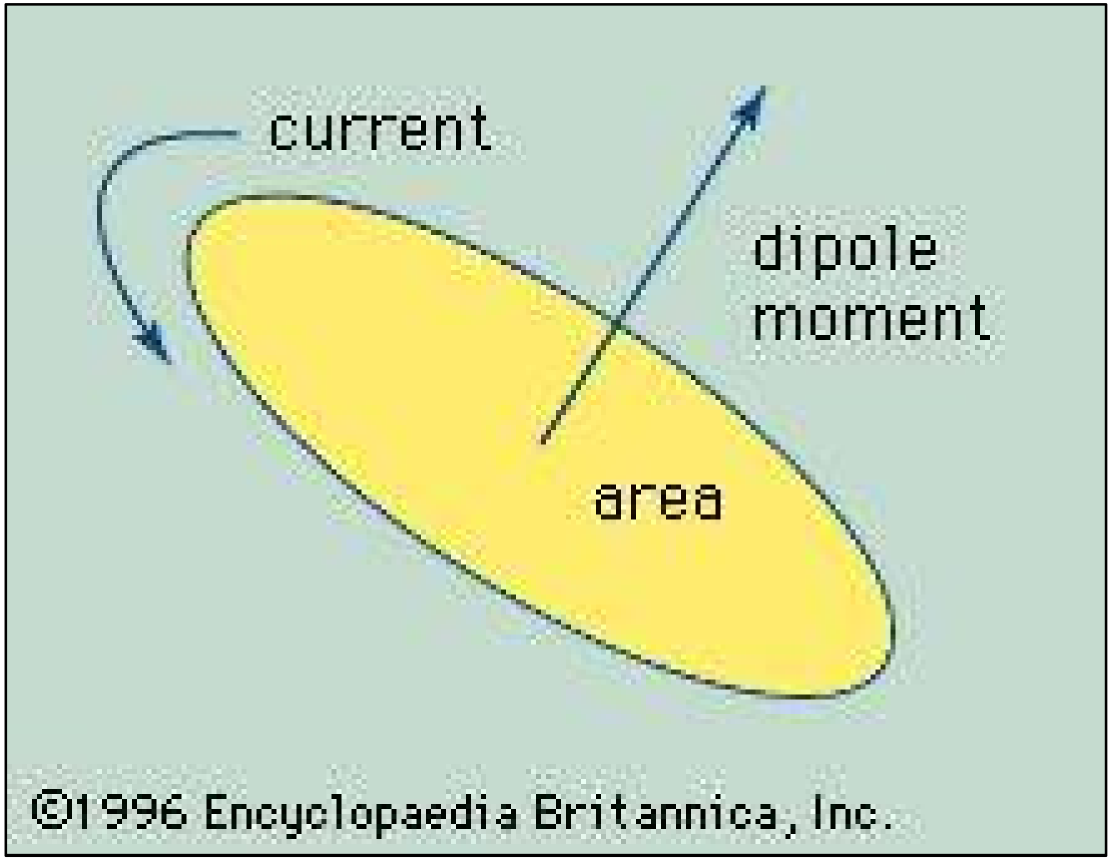
$$F_2 = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} l_2 \quad \text{(20-7)}$$



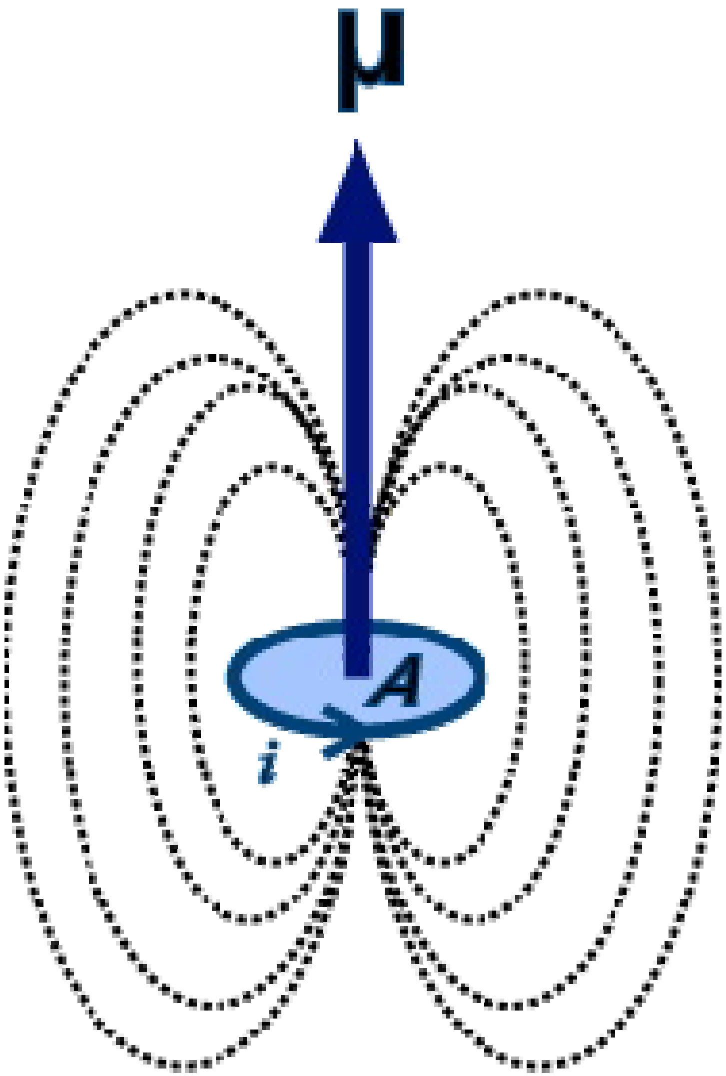
Magnetic Dipole



Circular Current is a Magnetic Dipole

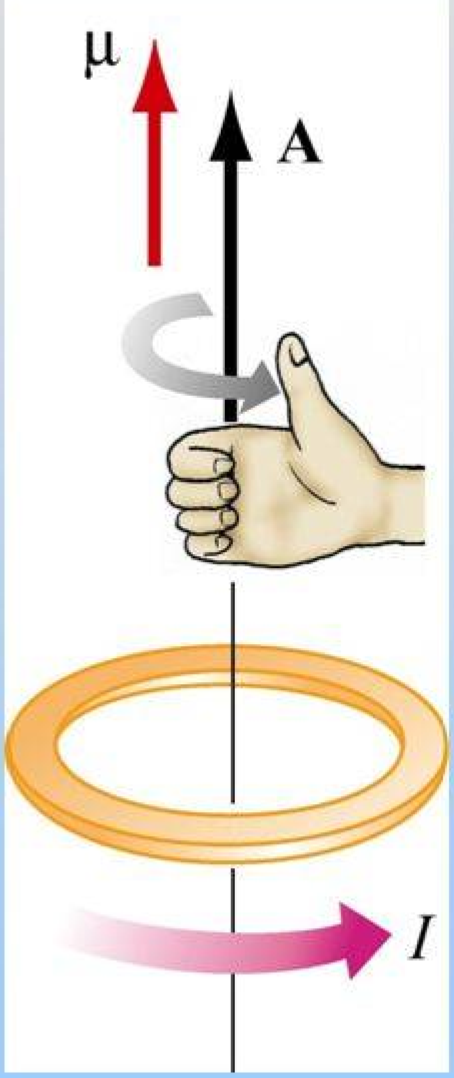


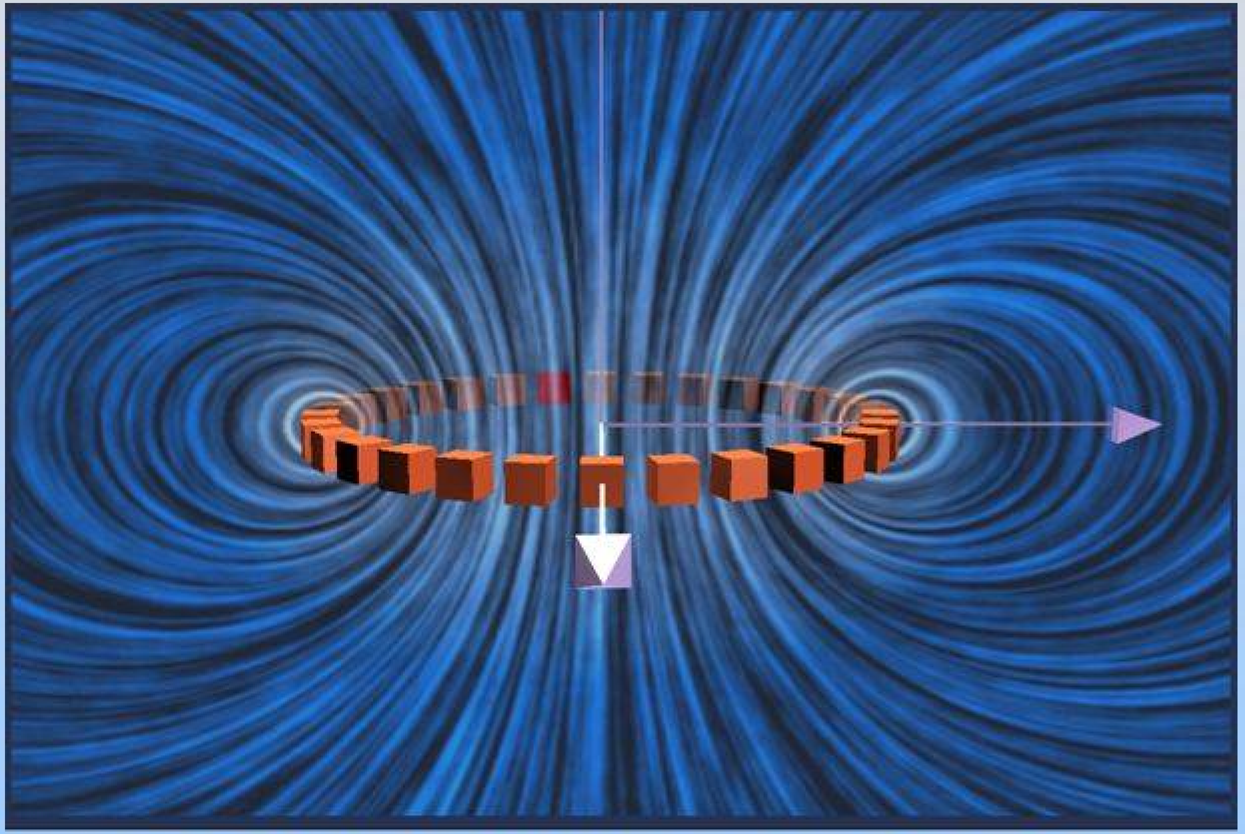
Circular Current is a Magnetic Dipole



Magnetic Dipole Moment

$$\vec{\mu} \equiv IA\,\hat{\mathbf{n}} \equiv I\vec{\mathbf{A}}$$





http://web.mit.edu/viz/EM/visualizations/magnetostatics/calculatingMagneticFields/RingMagField/RingMagField.htm

Magnetic Dipole moment

For z>>R:

$B_L = (u_0/4\pi)(2AI/z^3)$

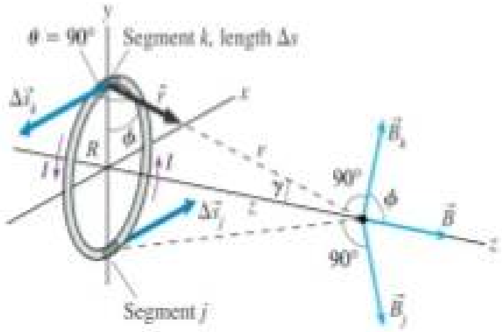
Where A is the area of the loop and I is the current.

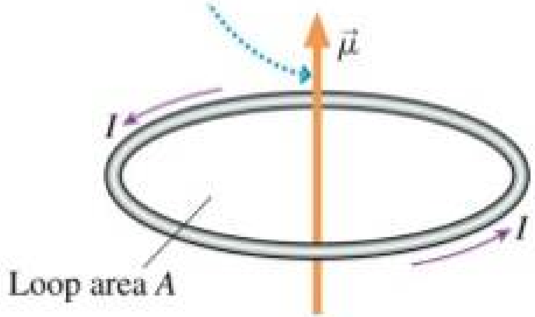
Define magnetic dipole moment:

$$u = (N)AI$$

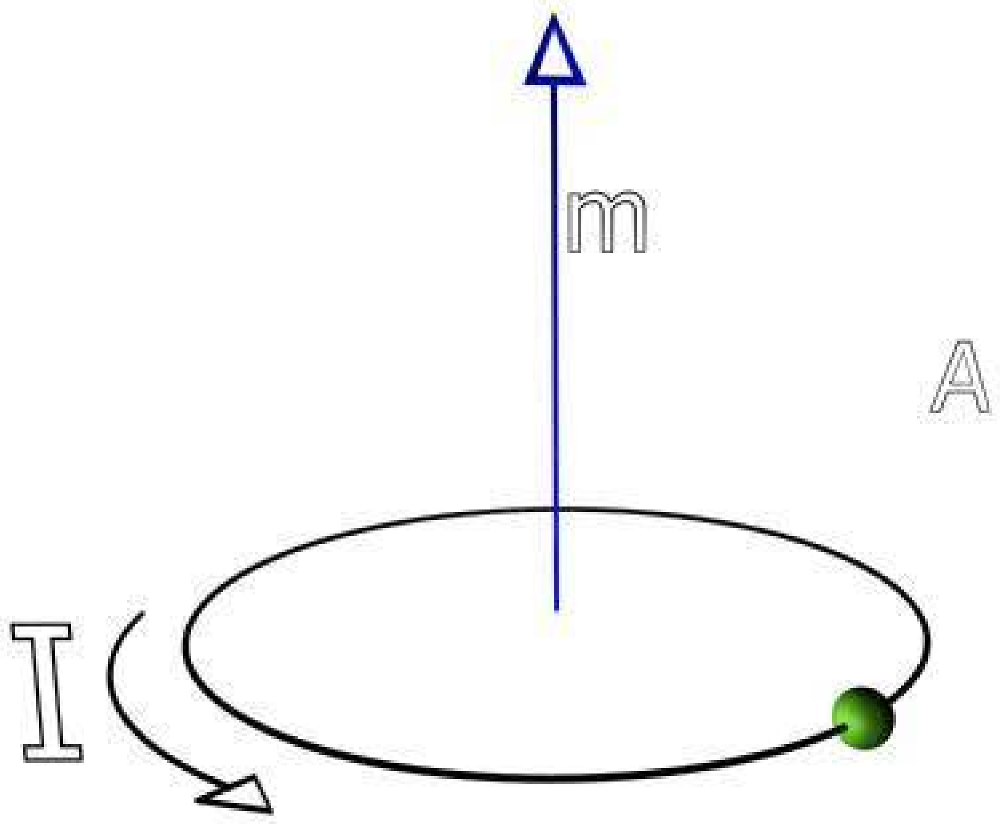
The quantity AI is what determines “how strong” the magnet is

Do not get u and u_0 confused in calculations.

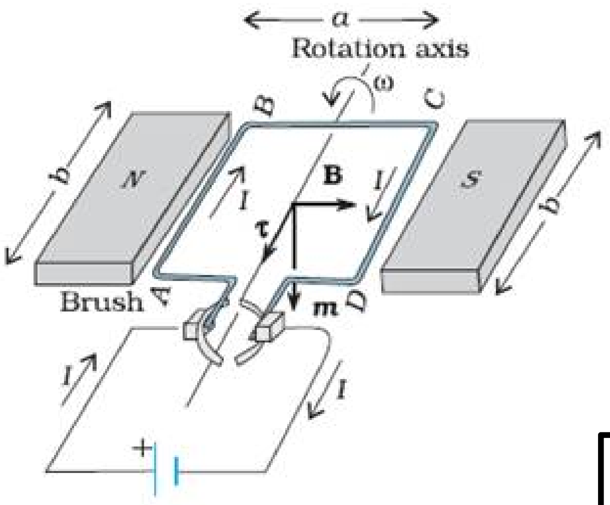




Magnetic Dipole Moment of a Revolving Electron



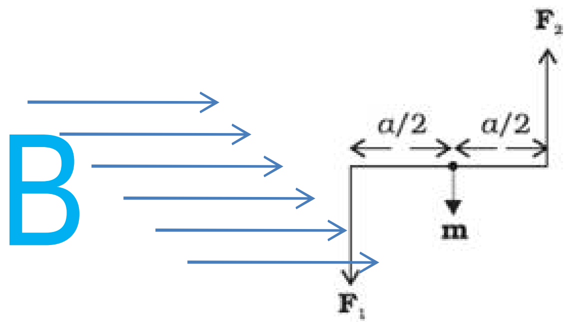
Magnetic Torque



(a)

FIGURE 4.21 (a) A rectangular current-carrying coil in uniform magnetic field. The magnetic moment **m** points downwards. The torque τ is along the axis and tends to rotate the coil anticlockwise. (b) The couple acting on the coil.

$$\tau = M \times B$$



(b)

M = **Current of Loop x Area of loop**
B = **Magnetic Field in the Region**

Torque on a Magnetic Dipole

Example 4.11 A 100 turn closely wound circular coil of radius 10 cm carries a current of 3.2 A. (a) What is the field at the centre of the coil? (b) What is the magnetic moment of this coil?

The coil is placed in a vertical plane and is free to rotate about a horizontal axis which coincides with its diameter. A uniform magnetic field of 2T in the horizontal direction exists such that initially the axis of the coil is in the direction of the field. The coil rotates through an angle of 90° under the influence of the magnetic field. (c) What are the magnitudes of the torques on the coil in the initial and final position? (d) What is the angular speed acquired by the coil when it has rotated by 90°? The moment of inertia of the coil is 0.1 kg m^2.

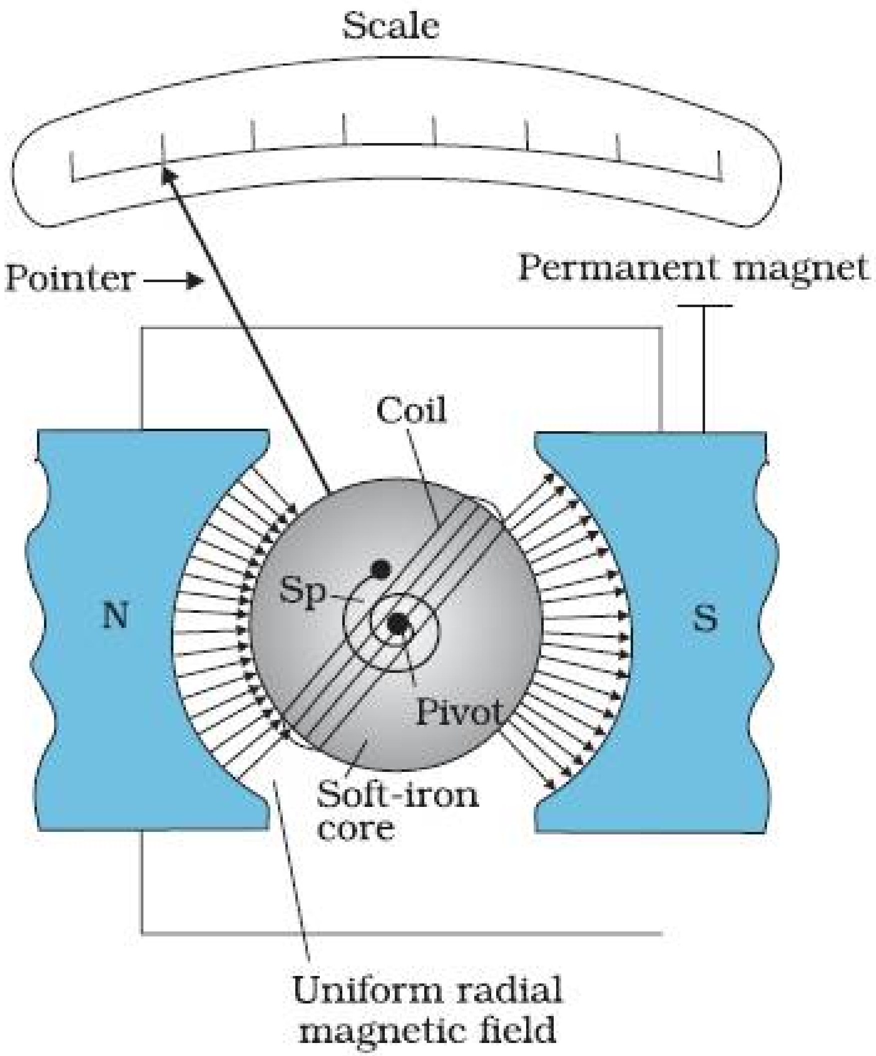
Example 4.12

(a) A current-carrying circular loop lies on a smooth horizontal plane. Can a uniform magnetic field be set up in such a manner that the loop turns around itself (i.e., turns about the vertical axis).

(b) A current-carrying circular loop is located in a uniform external magnetic field. If the loop is free to turn, what is its orientation of stable equilibrium? Show that in this orientation, the flux of the total field (external field + field produced by the loop) is maximum.

(c) A loop of irregular shape carrying current is located in an external magnetic field. If the wire is flexible, why does it change to a circular shape?

Moving Coil Galvanometer



Moving Coil Galvanometer